Topology II Homework 4

Problem 1: Show that $\mathbb{R}P^3$ is not homotopy equivalent to $\mathbb{R}P^2 \vee S^3$.

Problem 2: Show that if $\cup$ is a cup product, then then induced cross product $\times_\cup$ is bilinear and natural. Similarly, show that if $\times$ is a cross product, then the induced cup product $\cup_\times$ is bilinear and natural.

Problem 3: For this problem, let's go ahead and assume V,W, and Z are finite-dimensional vector spaces over $\mathbb{R}$. Show the following:

(a) $V \otimes W$ is naturally isomorphic to $W \otimes V$.
(b) $V \otimes (W \otimes Z)$ is naturally isomorphic to $(V \otimes W) \otimes Z$.
(c) $V^* \otimes W$ is naturally isomorphic to $\mathrm{Hom}(V, W)$.
(d) There are natural isomorphisms $(V \otimes W)^* = V^* \otimes W^* = \mathrm{Hom}(V, W^*) = \mathrm{Hom}(W, V^*)$.
(e) $\mathrm{Hom}(V \otimes W, Z)$ is naturally isomorphic to $\mathrm{Hom}(V, \mathrm{Hom}(W, Z))$.

Problem 4: In class, we showed that if (C, ∂) and (C', ∂') are chain complexes, then we could form a new chain complex by taking tensor products $(C \otimes C', \partial^\otimes)$. Show that this is indeed a chain comples (i.e., that $(\partial^\otimes)^2 = 0$).