Topology II Homework 5

Problem 1: Show that $\mathbb{R}P^3$ is not homotopy equivalent to $\mathbb{R}P^2 \vee S^3$.

Problem 2: Show that deleting a point from a manifold of dimension greater than 1 does not affect orientability of the manifold.

Problem 3: Show that every covering space of an orientable manifold is an orientable manifold.

Problem 4: For a map $f : M \to N$ between connected closed orientable n-manifolds with fundamental classes $[M]$ and $[N]$, the degree of f is defined to be the integer d such that $f_*([M]) = d[N]$, so the sign of the degree depends on the choice of fundamental classes. Show that for any connected closed orientable n-manifold M there is a degree 1 map $f : M^n \to S^n$.

Problem 5: Show that for a degree 1 map $f : M \to N$ of connected closed, orientable manifolds, the induced map $f_* : \pi_1(M) \to \pi_1(N)$ is surjective, hence also $f_* : H_1(M) \to H_1(N)$. [Hint: Lift f to the covering space $\widetilde{N} \to N$ corresponding to the subgroup $Im(f_*) \subset \pi_1(N)$, then consider the two cases that this covering is finite-sheeted or infinite-sheeted.]