Topology II Homework 4

Problem 1: If X is a CW complex that is the union of Y and Z where Y,Z, and $Y \cap Z$ are sub-complexes, then

$$\chi(X) = \chi(Y) + \chi(Z) - \chi(Y \cap Z)$$

Problem 2: Show that if X is a n-fold covering space of the CW complex X then $\chi(X) = n \cdot \chi(X)$.

Problem 3: Show that there is a continuous map $\mathbb{R}P^n \to \mathbb{R}P^m$ inducing a nontrivial map $H_1(\mathbb{R}P^m; \mathbb{Z}/2) \to H_1(\mathbb{R}P^n; \mathbb{Z}/2)$ if and only if $n \leq m$.

Problem 4: Show that any map $S^4 \to S^2 \times S^2$ must induce the zero map on H^4. Show that this is not necessarily true for maps $S^2 \times S^2 \to S^4$.

Problem 5: Let Σ_g be a surface of genus g. Let X be the wedge product of g copies of the torus T^2. Problem 1 in Section 3.2 of Hatchers book gives a quotient map $\Sigma_g \to X$. Compute the map induced on cohomology and determine the cup product structure on Σ_g in terms of the cup produce structure of T^2.

Problem 6: For finite CW complexes X and Y, show that $\chi(X \times Y) = \chi(X)\,\chi(Y)$.