# Topology II Homework 3

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**Problem 1:**(Pledged) Group the following spaces into homotopy equivalence classes. Justify your answers.

(a) Möbius band
(b) The torus
(c) $B^2 \times S^1$
(d) The torus minus one point
(e) The torus minus two points
(f) The Klein bottle minus one point
(g) $\mathbb{R}^3$ minus the $z$-axis
(h) $\mathbb{R}^3$ minus the unit circle in the $xy$-plane, $\{x^2 + y^2 = 0,\ z = 0\}$
(i) The intersection of (g) and (h)
(j) $S^3$ minus two linked circles
(k) $S^3$ minus two unlinked circles

**Problem 2:** Compute the homology of the CW complex obtained from the cube $I \times I \times I$ by identifying opposite faces after a 1/4 twist.

**Problem 3:** A map $f : S^n \to S^n$ satisfying $f(x) = f(-x)$ for all $x \in S^n$ is called an *even map*. Show that an even map $S^n \to S^n$ must have even degree, and that the degree must in fact be zero when $n$ is even. When $n$ is odd, show that there exist even maps of any given even degree. [Hint: First show that if $f$ is even, then it necessarily factors as a composition $S^n \to \mathbb{R}P^n \to S^n$.]

**Problem 4:** For an invertible linear transformation $S : \mathbb{R}^n \to \mathbb{R}^n$ show that the induced map on $H_n(\mathbb{R}^n, \mathbb{R}^n - \{0\}) \cong H_n(\mathbb{R}^{n-1} - \{0\}) \cong \mathbb{Z}$ is $Id$ or $-Id$ according to whether the determinant of $S$ is positive or negative. (Hint: Use Gaussian elimination to show that the matrix of $S$ can b joined by a path of invertible matrices to a diagonal matrix with $\pm 1$'s along the diagonal.)

**Problem 5:** Show that the quotient map $S^1 \times S^1 \to S^2$ obtained by collapsing the subspace $S^1 \vee S^1$ to a point is not nullhomotopic by showing that it induces an isomorphism on $H_2$. On the other hand, show via covering spaces that any map $S^2 \to S^1 \times S^1$ is nullhomotopic.

**Problem 6:** For finite CW complexes $X$ and $Y$, show that $\chi(X \times Y) = \chi(X)\,\chi(Y)$.