

Topology II Homework 2

Problem 1: Give a geometric description of the boundary map in the Mayer-Vietoris sequence.

Problem 2: Using the Mayer-Vietoris sequence, compute the homology of the n -Sphere, $H_*(S^n)$.

Problem 3: Let $T^2 = S^1 \times S^1$ be the torus, and $h : S^1 \rightarrow T^2$ an embedding of the unit circle into T^2 . Form the space

$$X = T^2 \cup_h D^2$$

by attaching a 2-cell D^2 to T^2 via the map h . Compute the homology of X . Note that there is more than one case.

Problem 4: Let X denote the surface of genus two with a single boundary component. Let A denote the boundary of X . Compute the relative homology groups $H_p(X, A)$.

Problem 5: Show that $S^1 \times S^1$ and $S^1 \vee S^1 \vee S^2$ have isomorphic homology groups, but that their universal covering spaces do not.