Topology I Homework 6

Problem 1: In the context of our proof of the zig-zag lemma. Prove that $\mathrm{Ker}(\phi_*) \subset \mathrm{Im}(\partial_*)$ and $\mathrm{Ker}(\psi_*) \subset \mathrm{Im}(\phi_*)$

Problem 2: Let $A : S^n \to S^n$ be the antipodal map. What is $A_* : \mathrm{H}_n \to \mathrm{H}_n(S^n)$?

Problem 3: Give a geometric description of the boundary map in the Mayer-Vietoris sequence.

Problem 4: Using the Mayer-Vietoris sequence, compute the homology of the n-Sphere, $\mathrm{H}_*(S^n)$.

Problem 5: Let $T^2 = S^1 \times S^1$ be the torus, and $h : S^1 \to T^2$ an embedding of the unit circle into T^2. Form the space

$$X = T^2 \cup_h D^2$$

by attaching a 2-cell D^2 to T^2 via the map h. Compute the homology of X. Note that there is more than one case.