Multivariable Analysis Homework 5

Problem 1: Let M be a nonempty closed smooth manifold (closed means $\partial M = \emptyset$). Show that there exists no smooth submersion $f : M \to \mathbb{R}^k$ for any $k > 0$.

Problem 2: Compute the flow of the following vector fields:
(1) $Z(x, y) = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}$ on $\mathbb{R}^2$.
(2) $Z(x, y) = (4x - 3y)\frac{\partial}{\partial x} + (6x - 5y)\frac{\partial}{\partial y}$ on $\mathbb{R}^2$.
(3) $Z(x) = (1 + x^2)\frac{\partial}{\partial x}$ on $\mathbb{R}$.
Why are the vector fields in (1) and (2) complete (even before computing the flow)? Draw a picture of each flow.

Problem 3: Show that $T(x, y, z) = (1 - z^2)y\frac{\partial}{\partial x} - (1 - z^2)x\frac{\partial}{\partial y}$ defines a vector field on $S^2 = \{(x, y, z) \in \mathbb{R}^3 \,|\, x^2 + y^2 + z^2 = 1\}$ and compute its flow on S^2. Can you describe it geometrically?

Problem 4: Let X be a vector field on M and $\gamma : I \to M$ an integral curve of X. Show that γ is the constant map if $\gamma'(t_0) = 0$ for some $t_0 \in I$.