

Topology I Homework 3

Problem 0: Please read Chapter 1 *The Fundamental Group* of Hatcher's book (pages 21 through 87). Pay especial attention to the examples discussed in this chapter.

Problem 1: Find a presentation of the Symmetric group on three symbols.

Problem 2: Show that $\langle a, b \mid a^4 = 1, b^2 = 1, bab^{-1} = a^{-1} \rangle$ is a presentation of a group of order 8 which is isomorphic to the group of symmetries (including flips) of the square.

Problem 3: Show that the groups

$$G = \langle a, b \mid a^2 = b^3 \rangle \quad \text{and} \quad H = \langle x, y \mid xyx = yxy \rangle$$

are isomorphic.

Problem 4: Show that the fundamental group of the Klein bottle has presentation

$$\langle a, b \mid aba^{-1}b = 1 \rangle.$$

Problem 5: Please do Problem 22 of Section 1.2 of Hatcher (on van Kampen's Theorem) concerning fundamental groups of knot complements.

Problem 6: Show that the fundamental group of the trefoil knot has a presentation

$$\langle x, y \mid xyx = yxy \rangle,$$

and show that this group is nonabelian. Conclude that the trefoil knot is not equivalent to the unknot.