Multivariable Analysis Homework 3

Problem 1: Prove that if $F : M \to N$ is a smooth map between smooth manifolds, then the coordinate representation of F is smooth with respect to *any* pair of charts on M and N.

Problem 2: Compute the coordinate representations for each of the following maps in stereographic coordinates, and use this to prove that each map is smooth.

(1) For each $n \in \mathbb{Z}$, the n^{th}*-power map* $p_n : S^1 \to S^1$, given in complex notation by $p_n(z) = z^n$.
(2) The antipotal map, $\alpha : S^n \to S^n$, given by $\alpha(x) = -x$.
(3) The Hopf fibration $H : S^3 \to S^2$, given by $H(z, w) = (z\overline{w}+w\overline{z}, iw\overline{z}-iz\overline{w}, z\overline{z}-w\overline{w})$, where we think of $S^3 \subset \mathbb{C}^2$ as the subset $\{(w, z)| \, |w|^2 + |z|^2 = 1\}$.

Problem 3: Suppose that $f : X \to Y$ is a diffeomorphism. Show that at each point $x \in X$, the map $Df_x : T_xX \to T_{f(x)}Y$ is an isomorphism.

Problem 4: Prove that $\mathbb{R}^k$ and $\mathbb{R}^\ell$ are diffeomorphic if and only if $k = \ell$.