Topology I Homework 2

Problem 0: If you have not finished yet, continue reading Chapters 2 and 3 of Munkres, paying especial attention to the following Sections: 2.16, 2.17, 2.18, 2.20, 2.22, 3.23, 3.26, 3.27. It is important that you be generally familiar with the material in these chapters/sections.

Problem 1: Show that a space X is simply-connected if and only if all paths having the same endpoints are fixed-endpoint homotopic. Note: the problem says "...paths having the same endpoints..." not "...loops having the same endpoints...".

Problem 2: Suppose that $f : (X, x_0) \to (Y, y_0)$ and $g : (Y, y_0) \to (Z, z_0)$ are continuous maps of pointed spaces. Show that $(g \circ f)_* = g_* \circ f_*$.

Problem 3: Let $p : E \to B$ be a covering map with $p(e_0) = b_0$. Let $F : [0, 1] \times [0, 1] \to B$ be continuous with $p(0, 0) = b_0$. Show that there is a lifting of F to a continuous map $F' : [0, 1] \times [0, 1] \to E$ such that $F'(0, 0) = e_0$. Show that if F is a path-homotopy, then so is F'.

Problem 4: Let $p : E \to B$ be a covering map, and suppose that B is connected. Show that if $p^{-1}(b)$ has k-elements for some $b \in B$, then it has k-elements for every $b' \in B$. In this case, we say that $p : E \to B$ is a *k-fold covering map.*

Problem 5: Show that if B is simply-connected, then any covering map $p : E \to B$ for which E is path-connected is a homeomorphism.

Problem 6: Let $h : (X, x_0) \to (Y, y_0)$ be a map. Show that if h is inessential, then

$$h_* : \pi_1(X, x_0) \to \pi_1(Y, y_0)$$

is trivial.