Topology I Homework 1

Problem 0: Read Chapters 2 and 3 of Munkres, paying especial attention to the following Sections: 2.16, 2.17, 2.18, 2.20, 2.22, 3.23, 3.26, 3.27.

Problem 1: Construct several examples of homotopic and non-homotopic maps. No proofs required.

Problem 2: Show that the relation of fixed-endpoint homotopy is an equivalence relation.

Problem 3: Construct some examples of paths which are fixed-endpoint homotopic, and some which are not. No proofs required.

Problem 4: A space X is *contractible* if the identity map $\mathrm{Id}_X : X \to X$ is homotopic to a constant map.

(a) Show that any convex open subset of $\mathbb{R}^n$ is contractible.
(b) Show that a contractible space is path connected.
(c) Show that if Y is contractible, then all maps

$$f : X \to Y$$

are homotopic.
(d) Show that if X is contractible and Y is path-connected, then all maps

$$f : X \to Y$$

are homotopic. What happens if we remove the path-connectedness assumption?

Problem 5: Check that the fundamental group of a space X (based at a point $x_0 \in X$) is indeed a group by verifying that the group axioms are satisfied.

Problem 6: In class, we showed that if x_0 and x_1 are points in the same path-component of a space X, then $\pi_1(X, x_0) \simeq \pi_1(X, x_1)$. Fill in the missing details from this proof.