Differential Geometry Homework 7

Problem 1: (Pledged) Show that Stokes's theorem from vector calculus is a special case of the Stokes's theorem for manifolds.

Problem 2: (Pledged) Let M be a closed, oriented manifold. Suppose that ω is a closed 1–form on M such that $\int_\gamma \omega = 0$ for every closed curve γ. Show that ω is exact.

Problem 3: (Pledged) Let X be a nowhere zero vector field on a closed manifold M. Show that there is a diffeomorphism $\phi : M \to M$ with no fixed points.

Problem 4: (pledged) Suppose that M is a compact, orientable n–dimensional manifold. Show that if ω is an $(n-1)$–form, then $d\omega$ vanishes at at least one point.

Problem 5: This question has two parts.

(a) If M is non-orientable and p is a point in M, is $M \setminus \{p\}$ be orientable?

(b) Let $f : M \to N$ be a local diffeomorphism. If one of M or N is orientable, must the other be as well?

Problem 6: For a smooth manifold M, show that T^*M is trivial as a vector bundle if and only if TM is trivial.

Problem 7: Show that the following 1–form on $\mathbb{R}^2 \setminus \{0\}$ is closed but not exact:

$$\omega = \frac{x\,dy - y\,dx}{x^2 + y^2}.$$

Problem 8: Let M be a closed manifold and $f : M \to N$ a smooth map. Suppose that there exists an open set $U \subset N$ for which $f : f^{-1}(U) \to U$ is a diffeomorphism. Show that N is compact.