Differential Geometry Homework 3

Problem 1: Show that $Sl(n, \mathbb{R})$ is an embedded submanifold of $Gl(n, \mathbb{R})$.

Problem 2: State a chain rule for differentiable manifolds and prove it.

Problem 3: Show that for two vector bundles E and F over a smooth manifold M, the space $Hom(E, F)$, which at each point $p \in M$ consists of of linear maps from E_p to F_p, is a vector bundle over M.

Problem 4: Show that there is no immersion $f : M^m \to \mathbb{R}^m$ if M is closed. (Closed means compact with no boundary.)

Problem 5 r: Let E be a vector bundle over B such that either B or the fibers of E are one-dimensional. Show that E is orientable if and only if E is trivial.