Our goal now is to prove the Dish theorem.

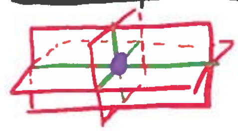
To do so, we need to better understand what a generic smooth map

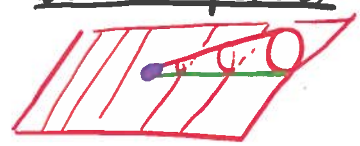
$$\Sigma \longrightarrow Y$$

looks like.

Fact: If $f: \Sigma \longrightarrow Y$ is a <u>generic smooth map</u>, then the singularities of f will consist of

① <u>Double-curves</u> ② <u>Triple-points</u> ③ <u>Branch-points</u>





From now on, we'll assume that all of our maps are generic.

Given a generic map $f: \Sigma \to Y$, let

$$S(f) = \{x \in \Sigma \mid f^{-1}(f(x)) \neq \{x\}\}$$

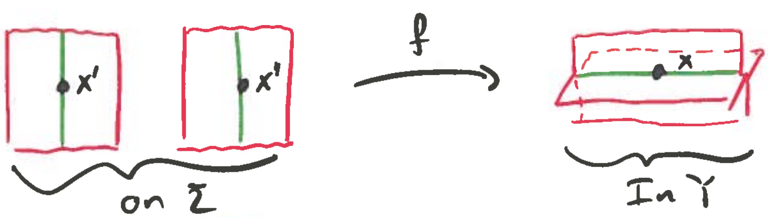
$$= \bigcup \left(\begin{array}{l}\text{immersed circles and properly} \\ \text{embedded arcs, all with } \wedge \\ \text{intersections + self-intersections}\end{array}\right)$$

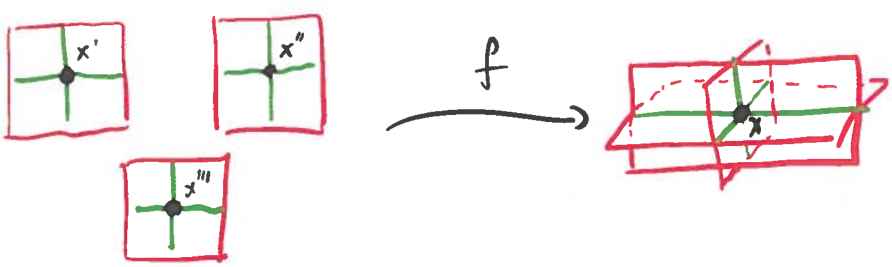
Also, let

$$\Sigma(f) = f(S(f)) \subset Y$$

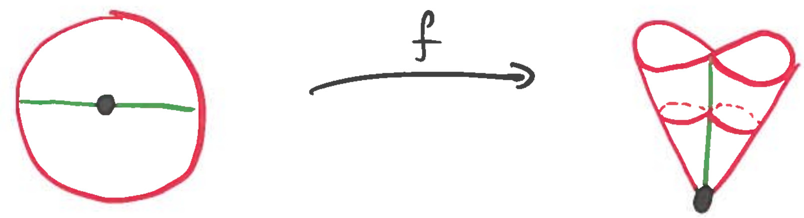
$$= \left(\begin{array}{l}\text{Union of curves} \\ \text{and arcs in } Y\end{array}\right)$$

<u>Pics</u>





②



Observe: If $f:(\Sigma,\partial\Sigma)\longrightarrow(Y,\partial Y)$, then we can assume $f(\mathrm{int}(\Sigma))\subset \mathrm{int}(Y)$

Pf

Basically, just add collar nbhds to Y and $\partial\Sigma$. The spaces stay the same.

Exercise: Given $f:\Sigma\longrightarrow Y$, with f an embedding near $\partial\Sigma$, show that $\exists$ a homotopic, but not necessarily smoothly homotopic map $\bar{f}:\Sigma\to Y$ with no branch points.

Def^{n} We call a double-curve simple if it is homeomorphic to S^1 (though it might intersect other double-curves).

Lemma: Let Y and f be as in the disk theorem, with f an embedding near ∂D. If $\Sigma(f)$ contains only simple double-curves, then the conclusion of the disk theorem holds.

pf: Let $\gamma\subset\Sigma(f)$

Then $f|_{f^{-1}(\gamma)}$ is a 2-1 covering map of S^1

Thus $f^{-1}(\gamma)=$  or 

In both cases, we have SCCs by assumption.

Let $N(\gamma)$ be a tubular nbhd of γ in Y. Since Y is orientable,

$$N(\gamma)\cong S^1\times D^2$$

③ pic

In this pic, the ends must be glued by a rotation of $0, \pi/2, \pi, 3\pi/2$

If $\boxed{\pi/2}$, then $\text{nbhd}(f^{-1}(\gamma))$ is two mobius bands ✗✗
$\boxed{\pi}$, " " " one " band ✗
$\boxed{3\pi/2}$, " " " two " bands ✗

Thus, the two ends must be glued without any rotation.

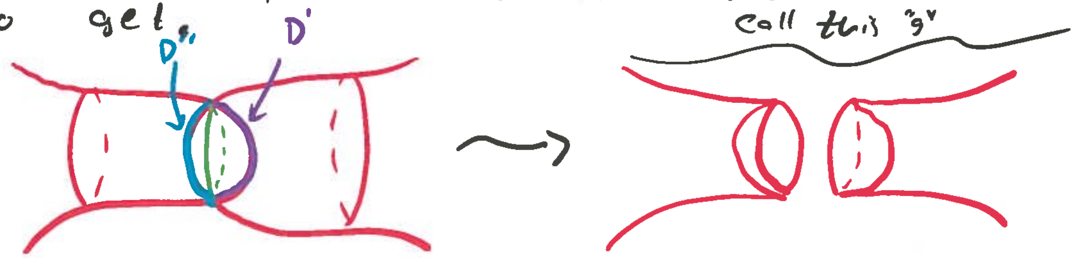
In particular,

$$f^{-1}(\gamma) \cong S^1 \amalg S^1$$

and each S^1 bounds a disk in D. Call them D' and D''.

Case 1: D' and D'' are disjoint in D

In this case, we surger f along γ to get

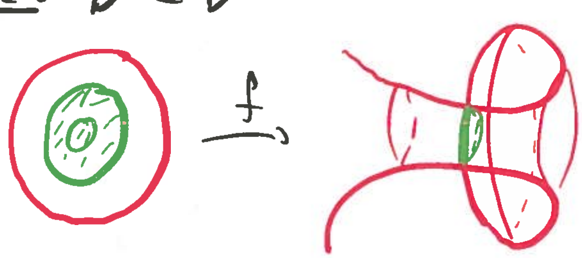


Then we get

$$g : (D^2, \partial D^2) \longrightarrow (Y, \partial Y)$$

with $g|_{\partial D^2} = f|_{\partial D^2}$

Case 2: $D' \subset D''$



In this case, form g by cutting out all the stuff in $D'' \setminus D'$