

Complex Analysis Homework 2

Problem 1: (*Type 1 problem: please provide both a rigorous solution to the problem and an intuitive explanation of the solution and why it's true.*)

With $w = se^{i\varphi}$, where $s \geq 0$ and $\varphi \in \mathbb{R}$, solve the equation $z^n = w$ in \mathbb{C} where $n \in \mathbb{Z}_{>0}$ is a natural number. How many solutions are there?

Problem 2: (*Pledged*)

Show that it is impossible to define a total ordering on \mathbb{C} . In other words, one cannot find a relation \succ between complex numbers so that:

- (i) For any two complex numbers z, w , one and only one of the following is true:
 $z \succ w$, $w \succ z$ or $z = w$.
- (ii) For all $z_1, z_2, z_3 \in \mathbb{C}$ the relation $z_1 \succ z_2$ implies $z_1 + z_3 \succ z_2 + z_3$.
- (iii) For all $z_1, z_2, z_3 \in \mathbb{C}$ with $z_3 \succ 0$ the relation $z_1 \succ z_2$ implies $z_1 z_3 \succ z_2 z_3$.

[Hint: First check if $i \succ 0$ is possible.]